

LESSON

4-1

Unit Rates**Practice and Problem Solving: A/B**

Solve.

1. To make 2 batches of nut bars, Jayda needs to use 4 eggs. How many eggs are used in each batch of nut bars?

2. On her way to visit her parents, Jennifer drives 265 miles in 5 hours. What is her average rate of speed in miles per hour?

3. Last week Alexander was paid \$56 for 7 hours of work. How much money does Alexander's job pay per hour?

4. Ned has scored 84 points in the first 6 games of the basketball season. How many points per game has Ned scored?

5. At the local grocery store, a 16-ounce bottle of apple juice costs \$3.20. What is the cost of the apple juice per ounce?

6. An above-ground swimming pool is leaking. After $\frac{1}{2}$ hour the pool has leaked $\frac{7}{8}$ of a gallon of water. How many gallons of water per hour is the swimming pool leaking?

7. After $\frac{3}{4}$ of a minute a sloth has moved just $\frac{3}{8}$ of a foot. What is the sloth's speed in feet per minute?

8. Food A contains 150 calories in $\frac{3}{4}$ of a serving. Food B contains 250 calories in $\frac{2}{3}$ of a serving. Find each unit rate. Which food has fewer calories per serving?

LESSON
4-1**Unit Rates*****Practice and Problem Solving: C*****Solve.**

1. Sasha can mow $\frac{3}{8}$ of an acre of grass in 45 minutes. How many acres of grass does Sasha mow per hour?

2. Ammar hikes $2\frac{3}{4}$ miles of nature trail in 1 hour and 15 minutes. How many miles of trail does Ammar hike per hour?

3. Melinda paints $\frac{7}{8}$ of a wall in $1\frac{1}{6}$ hours. What part of a wall does Melinda paint in 1 minute?

4. There is $\frac{1}{4}$ ounce of yeast in every $2\frac{1}{4}$ teaspoons of yeast. A recipe for bread calls for 2 teaspoons of yeast. How many ounces of yeast are needed for this recipe?

5. Every $5\frac{1}{2}$ cups of flour weighs $1\frac{9}{16}$ pounds. Use a unit rate to show how you could determine if there are more than or less than 35 cups of flour in a 10-pound bag of flour.

6. One tank is filling at a rate of $\frac{5}{8}$ gallon per $\frac{7}{10}$ hour. A second tank is filling at rate of $\frac{5}{9}$ gallon per $\frac{2}{3}$ hour. Which tank is filling faster? Explain how you know.

LESSON

4-1

Unit Rates

Practice and Problem Solving: D

Solve. The first one is done for you.

1. To make 2 loaves of banana bread, Leandra needs 6 eggs.
How many eggs are needed to make 1 loaf of banana bread?

$$\frac{6 \text{ eggs}}{2 \text{ loaves}} = \frac{3 \text{ eggs}}{1 \text{ loaf}}$$

Leandra needs 3 eggs to make 1 loaf of banana bread.

2. On his way to visit his sister at college, Gregg drives 135 miles in 3 hours. What is his average rate of speed in miles per hour?

$$\frac{135 \text{ miles}}{3 \text{ hours}} = \frac{\text{miles}}{1 \text{ hour}}$$

Gregg's average rate of speed is _____ miles per hour.

3. Jan designs a new logo for Kim's website. Kim pays Jan \$45 for 5 hours of work. How much money does Kim pay Jan per hour?

4. At a discount grocery store, Jessica paid \$0.72 for an 8-ounce bottle of spring water. What is the cost of the spring water per ounce?

5. A bucket is leaking. After 3 hours the bucket has leaked $\frac{3}{4}$ of an ounce. How many ounces per hour is the bucket leaking?

$$\frac{\frac{3}{4} \text{ oz}}{3 \text{ h}} = \frac{3}{4} \div \frac{3}{1} = \frac{3}{4} \times \frac{1}{3} = \frac{\text{oz}}{1 \text{ h}}$$

6. After 15 minutes a train has moved $\frac{9}{2}$ miles toward its destination.
How many miles per minute is the train moving?

7. A snack that Reginald just bought has 150 calories in $\frac{3}{4}$ of a serving.
How many calories per serving is this?

$$\frac{150 \text{ cal}}{\frac{3}{4} \text{ serving}} = \frac{150}{1} \div \frac{3}{4} = \frac{150}{1} \times \frac{4}{3} = \frac{\text{cal}}{1 \text{ serving}}$$

LESSON

4-1

Unit Rates

Reteach

A **rate** is a ratio that compares two *different* kinds of quantities or measurements.

3 aides for 24 students

$$\frac{3 \text{ aides}}{24 \text{ students}}$$

135 words in 3 minutes

$$\frac{135 \text{ words}}{3 \text{ minutes}}$$

7 ads per 4 pages

$$\frac{7 \text{ ads}}{4 \text{ pages}}$$

Express each comparison as a rate in ratio form.

1. 70 students per 2 teachers

2. 3 books in 2 months

3. \$52 for 4 hours of work

In a **unit rate**, the quantity in the denominator is 1.

300 miles in 6 hours

$$\frac{300 \text{ miles}}{6 \text{ hours}} = \frac{300 \div 6}{6 \div 6} = \frac{50 \text{ miles}}{1 \text{ hour}}$$

275 square feet in 25 minutes

$$\frac{275 \text{ ft}^2}{25 \text{ min}} = \frac{275 \div 25}{25 \div 25} = \frac{11 \text{ ft}^2}{1 \text{ min}}$$

Express each comparison as a unit rate. Show your work.

4. 28 patients for 2 nurses _____

5. 5 quarts for every 2 pounds _____

When one or both of the quantities being compared is a fraction, the rate is expressed as a **complex fraction**. Unit rates can be used to simplify rates containing fractions.

15 miles every $\frac{1}{2}$ hour

$$\frac{15 \text{ miles}}{\frac{1}{2} \text{ hour}} = 15 \div \frac{1}{2} = \frac{15}{1} \times \frac{2}{1} = \frac{30 \text{ miles}}{1 \text{ hour}}$$

 $\frac{1}{4}$ cup for every $\frac{2}{3}$ minute

$$\frac{\frac{1}{4} \text{ c}}{\frac{2}{3} \text{ min}} = \frac{1}{4} \div \frac{2}{3} = \frac{1}{4} \times \frac{3}{2} = \frac{3}{8} \text{ c}$$

Complete to find each unit rate. Show your work.

6. 3 ounces for every $\frac{3}{4}$ cup7. $3\frac{2}{3}$ feet per $\frac{11}{60}$ hour

LESSON
4-1

Unit Rates

Reading Strategies: Build Vocabulary

A **rate** is a special ratio that compares two values that are measured in different units. When one or both quantities being compared are fractions, the rate is expressed as a **complex fraction**.

\$8 for 2 pounds of beef

$$\begin{array}{c} \downarrow \\ \frac{\$8}{2 \text{ lb}} \\ \downarrow \end{array}$$

pounds compared to dollars

3 miles in $\frac{1}{4}$ hour

$$\begin{array}{c} \downarrow \\ \frac{3 \text{ mi}}{\frac{1}{4} \text{ h}} \\ \downarrow \end{array}$$

miles compared to hours

Answer each question.

1. Is the ratio $\frac{5 \text{ hours}}{12 \text{ hours}}$ a rate? Explain.

2. Is the ratio $\frac{50 \text{ yards}}{18 \text{ seconds}}$ a rate? Explain.

3. What does the rate $\frac{25 \text{ miles}}{\frac{2}{3} \text{ gallon}}$ compare?

In a **unit rate**, the second quantity in the rate is 1 unit. To rewrite a rate as a unit rate, rewrite the rate as a fraction with a denominator of 1.

Write yes or no to tell whether each rate is a unit rate. If a rate is not a unit rate, rewrite it as a unit rate.

4. $\frac{\$2.75}{1 \text{ h}}$ _____

5. $\frac{100 \text{ mi}}{4 \text{ gal}}$ _____

6. $\frac{200 \text{ ft}^2}{\frac{1}{4} \text{ h}}$ _____

7. $\frac{\frac{2}{3} \text{ lb}}{15 \text{ min}}$ _____

LESSON
4-1

Unit Rates

Success for English Learners

Problem 1

The Lawsons drive 288 miles in 6 hours. What is their average speed per hour?

Write the rate as a fraction.

$$\frac{\text{number of miles}}{\text{number of hours}} = \frac{\mathbf{288 \text{ miles}}}{\mathbf{6 \text{ hours}}}$$

Think: What can I do to change the denominator to a 1?

$$\frac{\mathbf{288 \text{ miles}}}{\mathbf{6 \text{ hours}}} \div \frac{\mathbf{6}}{\mathbf{6}} = \frac{\mathbf{48}}{\mathbf{1}}$$

$$288 \div 6 = 48$$



The unit rate is:

$$48 \text{ miles per hour} = \frac{\mathbf{48 \text{ miles}}}{\mathbf{1 \text{ hour}}}$$

So, their average speed is 48 miles per hour.

Problem 2

Bucket A is filling at a rate of $\frac{1}{2}$ cup per $\frac{2}{3}$ minutes. Bucket B is filling at a rate of $\frac{1}{3}$ cup per $\frac{1}{4}$ minutes. Which bucket is filling faster?

Write each rate as a fraction.



Bucket A

$$\frac{\mathbf{1 \text{ c}}}{\mathbf{\frac{2}{3} \text{ min}}}$$

Bucket B

$$\frac{\mathbf{1 \text{ c}}}{\mathbf{\frac{1}{4} \text{ min}}}$$



Rewrite each fraction as division, then solve.

Bucket A

$$\begin{aligned} \frac{\frac{1}{2}}{\frac{2}{3}} &= \frac{1}{2} \div \frac{2}{3} \\ &= \frac{1}{2} \times \frac{3}{2} \\ &= \frac{3}{4} \text{ c per min} \end{aligned}$$

Bucket B

$$\begin{aligned} \frac{\frac{1}{3}}{\frac{1}{4}} &= \frac{1}{3} \div \frac{1}{4} \\ &= \frac{1}{3} \times \frac{4}{1} \\ &= \frac{4}{3}, \text{ or } 1\frac{1}{3} \text{ c per min} \end{aligned}$$

Compare the unit rates.

Bucket A

$$\frac{3}{4}$$

<

Bucket B

$$1\frac{1}{3}$$

So, Bucket B is filling faster.

Solve.

- Seth walks $1\frac{1}{4}$ miles in $\frac{5}{12}$ hour. What is Seth's unit rate? _____
- Briana walks $\frac{3}{4}$ mile in $\frac{1}{5}$ hour. What is Briana's unit rate? _____
- Compare the walking speeds of Seth and Briana from questions 1 and 2. Who has the faster speed per hour?

LESSON
4-2

Constant Rates of Change

Practice and Problem Solving: A/B

Use the table to determine whether the relationship is proportional. If so, write an equation for the relationship. Tell what each variable you used represents.

1.

Number of tickets	2	3	4	5
Total Cost (\$)	54	81	108	135

- a. Proportional? _____
- b. Equation: _____
- c. Number of tickets: _____
- d. Total Cost: _____

2.

Weight (lb)	4	5	46
Total Cost (\$)	17.40	21.75	200.10

- a. Proportional? _____
- b. Equation: _____
- c. Weight: _____
- d. Total cost: _____

3.

Time (h)	2	3	4	5	6
Pages Read	50	75	90	110	120

4.

Time (h)	2	3	4
Distance (mi)	80	120	160

The tables show proportional relationships. Find the constant of proportionality, k . Write an equation to represent the relationship between the two quantities. Tell what each variable represents.

5.

Pens	3	6	9	12
Boxes	1	2	3	4

6.

Pack	1	2	4	5
Muffins	6	12	24	30

7. a. Create a table to show how the number of days is related to the number of hours. Show at least 5 days.
- b. Is the relationship proportional? _____
 - c. Write an equation for the relationship. _____

LESSON
4-2

Constant Rates of Change

Practice and Problem Solving: C

Answer the following questions.

1. Three tickets to attend an Off-Broadway show cost \$81, 4 tickets cost \$108, and 5 tickets cost \$135.
 - a. Show that the relationship between number and the cost is a proportional relationship by making a table of tickets for 1 to 5 tickets.

Number of Tickets					
Total Cost (\$)					

- b. The constant of proportionality k is _____.
 - c. Write an equation for the relationship: _____
2. On the seventh-grade trip to Washington, D.C., for every 8 students, there were 3 chaperones. Twelve chaperones were needed. How many students went on the trip?

Determine whether the relationship is a proportional relationship. If so, write an equation for the relationship, and tell what each of your variables represents. If the relationship is not proportional, explain.

3. Ty takes 1 hour to read 35 pages, 2 hours to read 70 pages, and 3 hours to read 105 pages.

4. There are 12 grams of protein in 2 ounces of almonds.

5.

Weight (lb)	4	5	6	7
Cost (\$)	18	22.5	27	31.5

6.

Time (h)	1	2	3	4
Distance (mi)	35	80	120	145

LESSON
4-2

Constant Rates of Change

Practice and Problem Solving: D

Use the table to determine whether the relationship is proportional. If so, write an equation to show the relationship between the two quantities. Tell what each of the variables you used represents. The first one has been done for you.

1.

Teams	1	2	3	4
Number of Players	6	12	18	24

- a. Proportional? yes
- b. Equation: $y = 6x$
- c. Number of teams: x
- d. Number of players: y

2.

Time (h)	1	2	3	4
Cars Washed	3	6	9	12

- a. Proportional? _____
- b. Equation: _____
- c. Number of hours: _____
- d. Cars washed: _____

3.

Weight (lb)	3	4	5
Cost (\$)	2.25	3.00	3.75

4.

Time (min)	2	3	4
Songs Played	10	14	20

The following tables show proportional relationships. Find the constant of proportionality, k . Then write an equation to show the relationship between the two quantities. Tell what each of the variables you used represents. The first one has been done for you.

5.

Apples	5	10	15	20
Bags	1	2	3	4

$k = \frac{1}{5}$

$y = \frac{1}{5}x;$

$x = \text{apples}; y = \text{bags}$

6.

Cartons	1	2	4	5
Eggs	12	24	48	60

$k =$ _____

LESSON
4-2

Constant Rates of Change

Reteach

A **proportion** is an equation or statement that two rates are the same.

*In 1 hour of babysitting, Rajiv makes \$8.
He makes \$16 in 2 hours, and \$24 in 3 hours.*

The same information is shown in the table below.

Time Worked (h)	1	2	3
Total Wage (\$)	8	16	24

To see if this relationship is proportional, find out if the rate of change is constant. Express each rate of change shown in the table as a fraction.

$$\frac{8}{1} = 8$$

$$\frac{16}{2} = 8$$

$$\frac{24}{3} = 8$$

The rate of change for each column is the same. Because the rate of change is constant, the relationship is *proportional*.

You can express a proportional relationship by using the equation $y = kx$, where k represents the constant rate of change between x and y .

In this example: $k = 8$. Write the equation as $y = 8x$.

The table shows the number of texts Terri received in certain periods of time.

Time (min)	1	2	3	4
Number of Texts	3	6	9	12

- Is the relationship between number of texts and time a proportional relationship? _____
- For each column of the table, write a fraction and find k , the constant of proportionality.

- Express this relationship in the form of an equation: _____
- What is the rate of change? _____

Write the equation for each table. Let x be time or weight.

5.

Time (h)	1	2	3	4
Distance (mi)	35	70	105	140

6.

Weight (lb)	3	4	5	6
Cost (\$)	21	28	35	42

LESSON
4-2

Constant Rates of Change

Reading Strategies: Use Graphic Aids

A **proportion** is a statement where two rates of change, or ratios, are **equivalent**. The statement below is one example of a proportional statement.

*For 1 hour of garden work, Kathy makes \$7.
She makes \$14 in 2 hours, and \$21 in 3 hours.*

You can use a table to help you see whether the relationship between two quantities is proportional. The table below shows the relationship between the number of hours Kathy works and the amount she is paid.

Time Worked (hr)	1	2	3
Total Wages (\$)	7	14	21

To tell whether the relationship between Kathy's time worked and her total wages is proportional, take the numbers in each column of the table, and write each pair as a ratio in the form of a fraction. Use time as your denominator and total wages as your numerator.

$$\frac{7}{1} = 7 \qquad \frac{14}{2} = 7 \qquad \frac{21}{3} = 7$$

All three fractions are equivalent, so the relationship is proportional.

The table shows the number of lengths of a swimming pool that Mario swam in certain time periods.

Time (min)	1	2	3	4
Distance (lengths)	3	6	9	12

- Find the rate at which Mario swam for each time period. Write the pair of numbers in each column of the table as a ratio in the form of a fraction. Use distance as the numerators and time as the denominators.

- What is the constant rate of change? _____
- Is the relationship between distance and time a proportional relationship? _____

Find the rate of change shown in each table.

4.

Time (h)	1	2	3	4
Distance (mi)	35	70	105	140

5.

Weight (lb)	4	5	6	7
Cost (\$)	17.40	21.75	26.10	30.45

LESSON
4-2

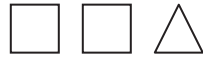
Constant Rates of Change

Success for English Learners

Problem 1

Write a proportion.

2 of the 3 objects are squares.

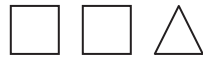


$$\frac{2}{3}$$



$$\frac{2}{3} = \frac{4}{6}$$

If there are 2 sets of objects, 4 will be squares.



$$\frac{4}{6}$$



$$\frac{2}{3} = \frac{4}{6}$$

How many would you have, if you had 4 ?

	1	2	3	4
	2	4	6	8

For every 2 squares, there is 1 triangle. The relationship is proportional.

Answer the following questions. First, complete the table below. Then, find the proportional relationship.

1. There are 6 for every 2 . Complete the table below to find the proportional relationship.

	6			12	15
	2	1	3		

2. For every , there are _____ .

LESSON
4-3

Proportional Relationships and Graphs

Practice and Problem Solving: A/B

Complete each table. Explain why the relationship is a proportional relationship.

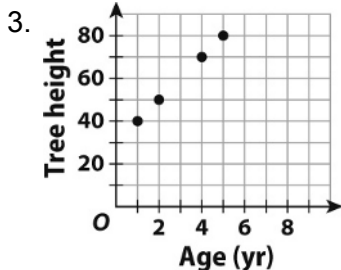
1. A cashier earns \$8 per hour.

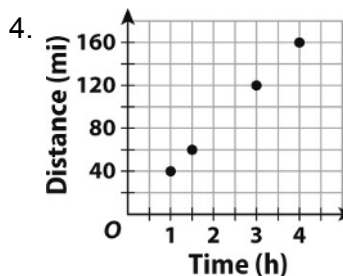
Time (h)	2	4		
Pay (\$)	16		40	72

2. Tomatoes cost \$0.70 per pound.

Weight (lb)	2		6	8
Price (\$)	1.40	2.10		

Tell whether the relationship is a proportional relationship. Explain your answer.

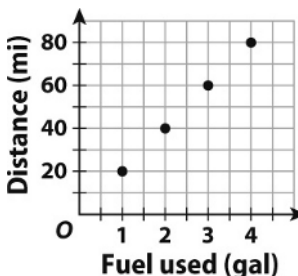




The graph shows the relationship between the distance traveled by a car and the amount of fuel used by the car.

5. Explain the meaning of (2, 40).

6. Write an equation for this relationship.



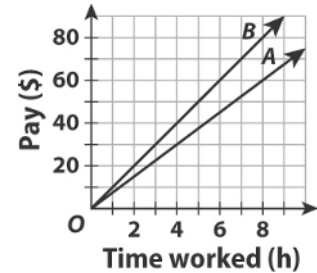
7. Suppose a compact car uses 1 gallon of fuel for every 27 miles traveled. How would the graph for the compact car compare to the graph for the car shown?

LESSON
4-3

Proportional Relationships and Graphs

Practice and Problem Solving: C

The graph shows the relationship between hours worked and money earned (in dollars) for two employees, A and B.



1. Suppose both employees work the same amount of time. Determine which employee earns more money. Explain.

2. Using the pay rates shown, determine the amount of money each employee earns for 15 hours of work.

3. The pay rate for employee C is less than the pay rate for employee B and greater than the pay rate for employee A. Write an equation for the possible pay y in dollars that employee C earns working x hours.

4. Two companies offer digital cable television as described below.

Company A: \$39.99 per month with no installation fee

Company B: \$34.99 per month with a \$50 installation fee

For each company, tell whether the relationship between months of service and total cost is a proportional relationship. Explain why or why not.

The table shows the relationship between the length and width of 5 different U.S. flags.

Width (ft), x	1.5	4.5	8	10.5	12.5
Length (ft), y	3	9	16	21	25

5. Is the relationship is a proportional relationship? If so, write an equation of the form $y = kx$ for the relationship.

6. Explain how to determine whether a relationship shown in a table is a proportional relationship.

LESSON
4-3

Proportional Relationships and Graphs

Practice and Problem Solving: D

Tell whether the relationship is a proportional relationship. Explain your answer. The first one is done for you.

1. Each shirt costs \$10.

Shirts	1	2	3	4
Cost (\$)	10	20	30	40

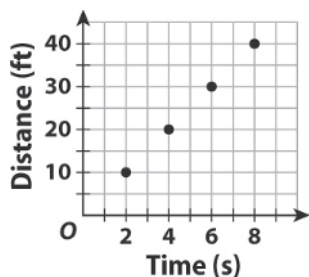
proportional; The cost is always

10 times the number of shirts.

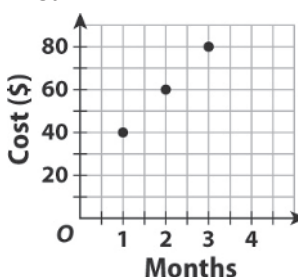
2. There are 50 crayons in each box.

Boxes of crayons	1	2	3
Crayons	50	100	150

3. A person walks 5 feet per second.



4. A gym costs \$20 per month plus a fee.



Write an equation for the proportional relationship with the given constant of proportionality k . The first one is done for you.

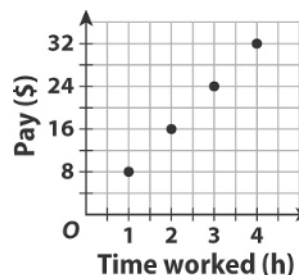
5. $k = 6$

$y = 6x$

6. $k = 4$

7. $k = \frac{1}{3}$

8. The graph shows the relationship between the money earned and the number of hours worked. Determine the constant of proportionality for this relationship. Show your work.



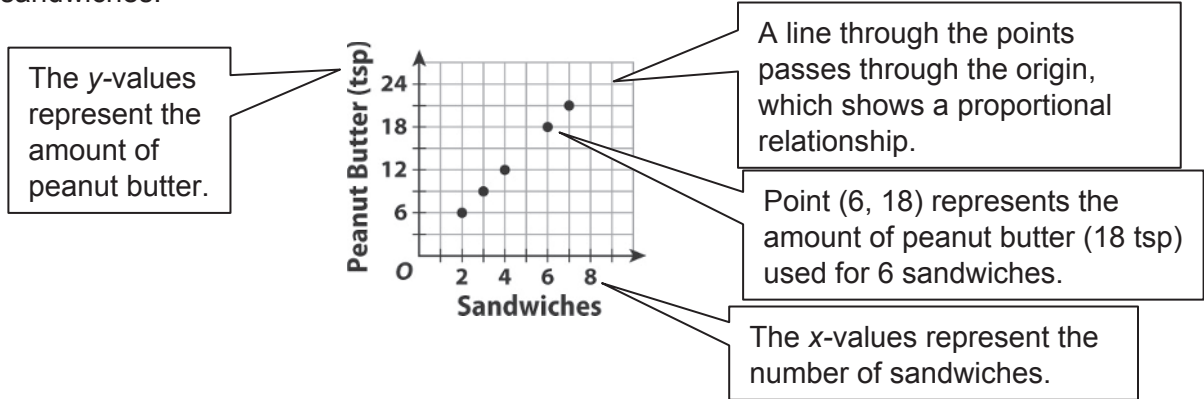
LESSON
4-3

Proportional Relationships and Graphs

Reteach

The graph of a proportional relationship is a line that passes through the origin. An equation of the form $y = kx$ represents a proportional relationship where k is the constant of proportionality.

The graph below shows the relationship between the number of peanut butter sandwiches and the teaspoons of peanut butter used for the sandwiches.

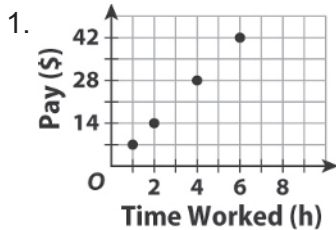


The constant of proportionality k is equal to y divided by x . Use the point $(6, 18)$ to find the constant of proportionality for the relationship above.

$$k = \frac{y}{x} = \frac{\text{amount of peanut butter}}{\text{number of sandwiches}} = \frac{18}{6} = 3$$

Using $k = 3$, an equation for the relationship is $y = 3x$.

Fill in the blanks to write an equation for the given proportional relationship.

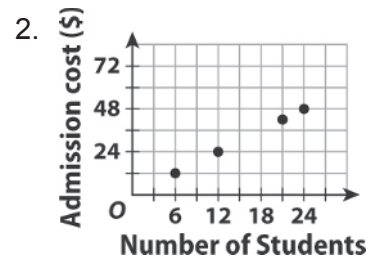


The x-values represent _____.

The y-values represent _____.

Using point _____, $k = \frac{y}{x} = \frac{\quad}{\quad} = \quad$.

An equation for the graph is _____.



The x-values represent _____.

The y-values represent _____.

Using point _____, $k = \frac{y}{x} = \frac{\quad}{\quad} = \quad$.

An equation for the graph is _____.

LESSON

4-3

Proportional Relationships and Graphs***Reading Strategies: Reading Tables and Graphs***

When two quantities are related by a constant multiplier, the quantities have a **proportional relationship**. The graph of a proportional relationship is a straight line through the origin.

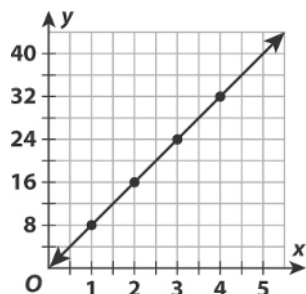
The table below shows the relationship between the number of glasses filled and the amount of juice needed to fill them. The relationship between these two quantities is a proportional relationship.

Glasses	1	2	3	4
Juice Needed (oz)	8	16	24	32

1. What are the quantities used to form this relationship?

2. How are the two quantities related?

The data from the table is shown in the graph below. Notice that a line drawn through the points passes through the origin.



3. What do the x-values in the graph represent?

4. What do the y-values in the graph represent?

5. What does the ordered pair (2, 16) represent?

6. Write an ordered pair for 3 glasses and the amount of juice needed.

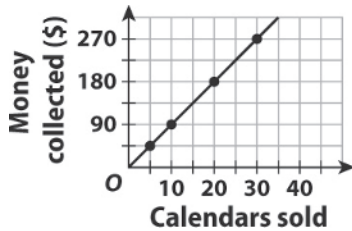
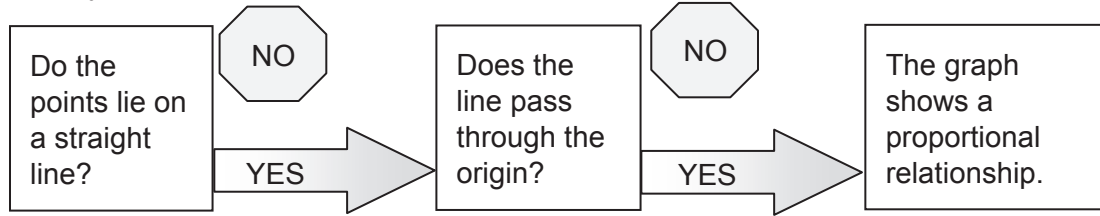
LESSON
4-3

Proportional Relationships and Graphs

Success for English Learners

Problem 1

Does the graph show a proportional relationship? Use a flowchart to help you.



Question	NO	YES
Points on straight line?		✓
Does line go through (0, 0)?		✓
Proportional relationship?		✓

Problem 2

Write an equation for the proportional relationship in the graph above.

1

Pick a point on the graph.

2

Find the constant of proportionality k .

3

Write an equation of the form $y = kx$.

(10, 90)

$$k = \frac{y}{x} = \frac{\text{money collected}}{\text{calendars sold}} = \frac{90}{10} = 9$$

$y = 9x$

1. Complete the following statements about the constant of proportionality k in a proportional relationship.

- constant of proportionality = $\frac{\text{change in } \square \text{ - values}}{\text{change in } \square \text{ - values}}$

- The constant of proportionality is the ratio of the _____ to the _____.

2. Do all straight lines represent proportional relationships? Explain.

MODULE
4

Ratios and Proportionality

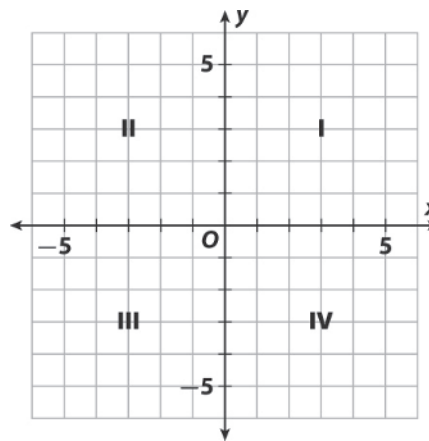
Challenge

Variations on Variation

An equation with the general form $y = kx$ represents a proportional relationship called **direct variation**. The variable y varies directly with x . Here are two other kinds of variation.

quadratic variation: $y = kx^2$ **indirect variation:** $y = \frac{k}{x}$

Examples with real-world contexts usually have values of k greater than 0. The graphs are often shown only in the first quadrant, quadrant I. But k doesn't have to be positive. For values of x less than 0, the graphs will be in more than one quadrant.



Use a graphing calculator or computer graphing program. For each problem, analyze all three variation equations: direct, quadratic, and indirect.

1. Let $k = 1$. Describe what the three graphs look like in the first quadrant.

direct: _____

quadratic: _____

indirect: _____

2. Let $k = 1$. Tell which quadrants contain the graphs for *all* values of the independent variable x .

direct: _____ quadratic: _____ indirect: _____

3. For each type of variation, describe what happens in the first quadrant as k gets larger than 1. Try several different values for k .

direct: _____

quadratic: _____

indirect: _____

4. Explore what happens when k is negative. Which quadrants contain the graphs?

direct: _____ quadratic: _____ indirect: _____

LESSON
5-1**Percent Increase and Decrease*****Practice and Problem Solving: A/B*****Find each percent increase. Round to the nearest percent.**

1. From 24 teachers to 30 teachers _____
2. From \$18 to \$45 _____
3. From 75 pencils to 225 pencils _____
4. From \$65 to \$144 _____
5. From 42 acres to 72 acres _____
6. From 95 trees to 145 trees _____

Find each percent decrease. Round to the nearest percent.

7. From 20 miles to 11 miles _____
8. From \$16 to \$4 _____
9. From 126 ounces to 48 ounces _____
10. From 84 seconds to 8 seconds _____
11. From 90 apples to 75 apples _____
12. From 248 workers to 200 workers _____

Given the original amount and the percent of change, find the new amount.

13. \$25; 300% increase _____
14. 160 bananas; 20% decrease _____
15. 56 books; 75% decrease _____
16. 52 companies; 25% increase _____
17. 12,000 miles; 5% increase _____
18. 710 points; 10% decrease _____

Solve.

19. Last year, there were 380 students at Woodland Middle School. This year, the student population will increase by 5%. What will be the school's increased student population?
- _____

20. A backpack that normally sells for \$39 is on sale for 33% off. Find the amount of the discount and the sale price.
- _____

21. In August, the Simons' water bill was \$48. In September, it was 15% lower. What was the Simons' water bill in September?
- _____

22. A gallery owner purchased a very old painting for \$3,000. The painting sells at a 325% increase in price. What is the retail price of the painting?
- _____

LESSON

5-1

Percent Increase and Decrease***Practice and Problem Solving: C*****Solve.**

- Enrollment in the school orchestra was 340 last year. This year, it dropped 15%.
 - What is the enrollment this year? _____
 - If enrollment increases 15% next year, what will the enrollment be?

Round to the nearest whole number. _____

- Rodrigo and Samantha work part-time selling magazine subscriptions. Their boss said that he wanted them to increase sales by 15% each week. As an incentive, whoever increases their sales by the higher percentage each week would get a bonus. The table below shows subscription sales for four weeks in a row.

Number of Subscriptions Sold

Worker	Week 1	Week 2	Week 3	Week 4
Rodrigo	17	25	18	27
Samantha	15	18	21	23

- Find Rodrigo's percent increase or decrease in sales each week.

 - Find Samantha's percent increase or decrease in sales each week.

 - Who received a bonus in weeks 2, 3, and 4?

 - After four weeks, did Rodrigo or Samantha meet their boss's goal? Explain.

- The Kelvin scale is an absolute temperature scale that can be used to calculate percent change in temperatures. The formula to convert Celsius to Kelvin is $K = C + 273.15$. What is the percent change if the temperature rises from 21°C to 29°C ? Round to the nearest tenth.

LESSON
5-1**Percent Increase and Decrease*****Practice and Problem Solving: D***

Find each percent increase. Round to the nearest percent. The first one is done for you.

1. From \$15 to \$21 40%
2. From 12 teachers to 48 teachers _____
3. From 80 pencils to 152 pencils _____
4. From 40 cans to 70 cans _____

Find each percent decrease. Round to the nearest percent. The first one is done for you.

5. From 80 miles to 15 miles 81%
6. From 100 ounces to 25 ounces _____
7. From \$60 to \$40 _____
8. From 39 seconds to 13 seconds _____

Find the new amount given the original amount and the percent of change. The first one is done for you.

9. \$25; 10% increase \$27.50
10. 160 bananas; 20% decrease _____
11. 200 books; 75% decrease _____
12. 52 companies; 25% increase _____

Solve.

13. Last year, there were 400 students at Woodland Middle School. This year, the student population will increase by 5%. What will be the school's student population this year?

14. A backpack that normally sells for \$39 is on sale for 30% off. Find the amount of the sale price.

LESSON

5-1

Percent Increase and Decrease**Reteach**

A change in a quantity is often described as a percent increase or percent decrease. To calculate a percent increase or decrease, use this equation.

$$\text{percent of change} = \frac{\text{amount of increase or decrease}}{\text{original amount}} \cdot 100$$

Find the percent of change from 28 to 42.

- First, find the amount of the change. $42 - 28 = 14$
- What is the original amount? 28
- Use the equation. $\frac{14}{28} \cdot 100 = 50\%$

An increase from 28 to 42 represents a 50% increase.

Find each percent of change.

1. 8 is increased to 22

amount of change: $22 - 8 = \underline{\hspace{2cm}}$

original amount: $\underline{\hspace{2cm}}$

$\underline{\hspace{2cm}} \cdot 100 = \underline{\hspace{2cm}}\%$

2. 90 is decreased to 81

amount of change: $90 - 81 = \underline{\hspace{2cm}}$

original amount: $\underline{\hspace{2cm}}$

$\underline{\hspace{2cm}} \cdot 100 = \underline{\hspace{2cm}}\%$

3. 125 is increased to 200

amount of change: $200 - 125 = \underline{\hspace{2cm}}$

original amount: $\underline{\hspace{2cm}}$

$\underline{\hspace{2cm}} \cdot 100 = \underline{\hspace{2cm}}\%$

4. 400 is decreased to 60

amount of change: $400 - 60 = \underline{\hspace{2cm}}$

original amount: $\underline{\hspace{2cm}}$

$\underline{\hspace{2cm}} \cdot 100 = \underline{\hspace{2cm}}\%$

5. 64 is decreased to 48

6. 140 is increased to 273

7. 30 is decreased to 6

8. 15 is increased to 21

9. 7 is increased to 21

10. 320 is decreased to 304

LESSON
5-1**Percent Increase and Decrease****Reading Strategies: Analyze Information**

Percent can be used to describe change. It is shown as a ratio.

$$\text{Percent of change} = \frac{\text{amount of change}}{\text{original amount}}$$

The following steps describe how the percent of change is figured on a savings account that starts with \$50.

Original amount in the account: \$50

Current amount in the account: \$30

Amount the account decreased by: \$20

$$\text{Percent of change} = \frac{\text{amount of change}}{\text{original amount}} = \frac{\$20}{\$50}$$

Savings went down, so this ratio is a **percent of decrease** in savings.

1. How much money was placed into the savings account when it opened? _____
2. Did the number of dollars in the account increase or decrease?

3. When you are finding the percent of change, where do you place the original number in the fraction?

Use the information below to solve 4–6.

A clothing salesman sold 25 shirts his first day on the job and 45 shirts the second day.

4. What is the original number of shirts he sold?

5. How many more shirts did he sell the second day than the first day?

6. Write the fraction that shows the amount of change over the original amount. What is the percent of change? Is it a percent increase or percent decrease?

LESSON
5-1

Percent Increase and Decrease

Success for English Learners

Problem 1

$$\text{percent of change} = \frac{\text{amount of change}}{\text{original amount}}$$

27 is **DECREASED** to 20

The new amount is less than the original amount, so **SUBTRACT** to find the amount of change

$$\text{percent decrease} = \frac{\text{original amount} - \text{new amount}}{\text{original amount}}$$

$$= \frac{27 - 20}{27}$$

$$= \frac{7}{27}$$

$$= 27 \overline{)7}$$

$$= 27 \overline{)7.000} \quad \begin{array}{r} 0.259 \end{array}$$

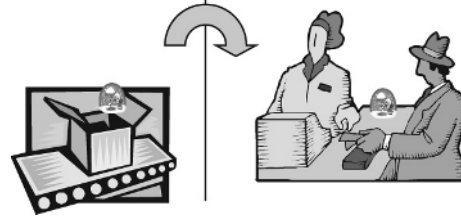
$$\approx 25.9\%$$

Problem 2

How do you find the retail price?

Buys snow globes for \$9.20.

Sells snow globes at a 95% increase.



Step 1

Find 95% of \$9.20 → $\$9.20 \times 0.95 = \8.74

Step 2

To get the retail price, add the amount from Step 1 to the original price, \$9.20.

$$\$9.20 + \$8.74 = \$17.94$$

1. Explain the difference between a percent increase and a percent decrease.

2. Give a real-life example of a percent increase.

3. Explain the difference between retail and wholesale.

4. Fill in the blank: retail = _____ + amount of increase

5. Write your own problem using percent of change.

LESSON
5-2**Rewriting Percent Expressions****Practice and Problem Solving: A/B**

Use the situation below to complete Exercises 1–6 in the table below.

Discounts R Us buys items at wholesale, then marks them up to set a retail sale price. Some of the items the store sells are shown in the table below.

Item	Wholesale Price	% Markup	\$ Markup	Retail Sale Price
1. Notebook	\$1.50	20%		
2. Scissors	\$3.25	40%		
3. Calculator	\$9.60	25%		
4. Sunglasses	\$12.50	78%		
5. Bicycle	\$78.00	55%		
6. Picture frame	\$2.99	150%		

Find the retail sale price of each item below. Round to two decimal places when necessary.

7. Original price: \$65.00; Markdown: 12%

8. Original price: \$29.99; Markdown: $33\frac{1}{3}\%$

9. Original price: \$119.00; Markdown: 70%

10. Original price: \$325.50; Markdown: 15%

Use the information to complete 11–14.

A jewelry supply shop buys silver chains from a manufacturer for c dollars each, and then sells the chains at a 57% markup.

11. Write the markup as a decimal. _____

12. Write an expression for the retail price of a silver chain.

13. What is the retail price of a silver chain purchased for \$45.00?

14. How much was added to the original price of the chain? _____

LESSON

5-2

Rewriting Percent Expressions***Practice and Problem Solving: C***

Answer the questions about each situation.

A clothing store offers various promotions to attract customers but wants to maintain the same amount of profit. To do this, the store marks up prices above retail and then advertises a deal that results in a markdown equivalent to the retail price. The retail price of a necktie is \$59.99. The store offers a “buy 2, get one free” deal on neckties.

1. What is the new retail price during this promotion? _____
2. What is the markup? _____
3. What is the markup percent on a tie during the promotion? _____

A sporting goods store sells jerseys with the name of the local football team. The store owner buys 80 jerseys at a wholesale price of \$55 each and applies his standard retail markup of 65%. When the local team made it to the playoffs, the store owner marked up the retail price by 25%. After the local team lost the championship, the store owner marked down that price by 33%.

4. What was the standard retail price of each jersey? _____
5. What was the price when the team was in the playoffs? _____
6. What was the price after the team lost the championship? _____

Two jewelry stores buy silver chains from a manufacturer for c dollars each, and then sell the chains at a 57% markup. Store A has a sale and marks down all chains by 20% off retail. In addition, customers can use a coupon worth 15% off the price of any item, including sale items. Store B offers a coupon worth 35% off any one item.

7. At Store A, Aurelie used a 15%-off coupon to buy a chain already marked down by 20%. Write an expression for the price of this chain.

8. At Store B, Tucker used a 35%-off coupon to buy a chain. Write an expression for the price of this chain.

9. Which store offers a better price on chains?

LESSON
5-2

Rewriting Percent Expressions

Practice and Problem Solving: D

Answer each question. The first one is done for you.

1. Abdul buys dress pants from a clothing company for p dollars. He then sells each pair of pants in his men's clothing shop at a 40% markup.

a. Write the markup as a decimal. 0.40p

b. Write an expression for the retail price of a pair of dress pants.

c. What is the retail price of the pants that Abdul purchased for \$56.00?

d. How much did Abdul add to the original price of the pants?

Complete the table. The first row is done for you.

Item	Price	% Markup	\$ Markup	Retail Price
2. Tie	\$30	20%	\$6.00	\$36.00
3. Cufflinks	\$10	35%		
4. Belt	\$40	25%		

Find the sale price of each item. Round to two decimal places when necessary. The first one is done for you.

5. Original price: \$65; Markdown: 10%

58.50

6. Original price: \$30.50; Markdown: 30%

7. Original price: \$105; Markdown: 75%

8. Original price: \$325; Markdown: 15%

9. A jewelry supply shop buys silver chains from a manufacturer for c dollars each, and then sells the chains at a 40% markup. Write an expression for the retail price of a silver chain.

LESSON
5-2

Rewriting Percent Expressions

Reteach

A **markup** is an example of a percent increase.

To calculate a markup, write the markup percentage as a decimal and add 1. Multiply by the original cost.

A store buys soccer balls from a supplier for \$5. The store's markup is 45%. Find the retail price.

Write the markup as a decimal and add 1.

$$0.45 + 1 = 1.45$$

Multiply by the original cost.

$$\text{Retail price} = \$5 \times 1.45 = \$7.25$$

A **markdown**, or discount, is an example of a percent decrease.

To calculate a markdown, write the markdown percentage as a decimal and subtract from 1. Multiply by the original price.

A store marks down sweaters by 20%. Find the sale price of a sweater originally priced at \$60.

Write the markup as a decimal and subtract it from 1.

$$1 - 0.2 = 0.8$$

Multiply by the original cost.

$$\text{Sale price} = \$60 \times 0.8 = \$48$$

Apply the markup for each item. Then, find the retail price. Round to two decimal places when necessary.

1. Original cost: \$45; Markup %: 20%

2. Original cost: \$7.50; Markup %: 50%

3. Original cost: \$1.25; Markup %: 80%

4. Original cost: \$62; Markup %: 35%

Apply the markdown for each item. Then, find the sale price. Round to two decimal places when necessary.

5. Original price: \$150; Markdown %: 40%

6. Original price: \$18.99; Markdown: 25%

7. Original price: \$95; Markdown: 10%

8. Original price: \$75; Markdown: 15%

9. A clothing store bought packages of three pairs of socks for \$1.75. The store owner marked up the price by 80%.

a. What is the retail price? _____

b. After a month, the store owner marks down the retail price by 20%.

What is the sales price? _____

LESSON
5-2

Rewriting Percent Expressions

Reading Strategies: Use a Model

A toy store buys kites from a supplier for k dollars. The store owner decides to mark up the price of each kite by 30%. What is the retail price of a kite that the supplier sells for \$60?

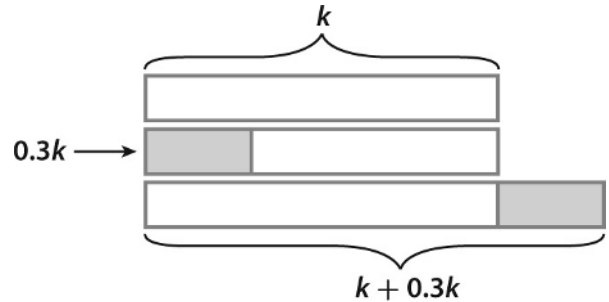
You can use a bar model to help you solve this problem.

Step 1: Draw a bar to show the cost of the kite, k .

Step 2: Draw a bar that shows the markup: 30% of k , or $0.3k$.

Step 3: These bars together represent the original cost plus the markup, $k + 0.3k$.

Step 4: Use the expression to find the retail price of a kite.



$$\begin{aligned} \text{Retail price} &= \text{Original cost} + \text{Markup} \\ &= k + 0.3k \\ &= 1.3k = 1.3(\$60) = \$78 \end{aligned}$$

The retail price of each kite is \$78.

Answer each question. As you do, draw each step of the bar model in the space at the right.

A store buys cameras from a supplier for c dollars. The store owner decides on a markup of 70%. What is the retail price of a camera that the supplier sells for \$80?

- When you use a bar model to solve the problem, which bar should you draw first?

- Which bar should you draw next?

- What do these bars together represent?

- What is the retail price of the camera?

LESSON
5-2

Rewriting Percent Expressions

Success for English Learners

A **markup** is a percent *increase*. With a markup, the price goes **UP**. ↑

A **markdown** is a percent *decrease*. With a markdown, the price goes **DOWN**. ↓

Problem 1

Find the retail price.

$$\text{Retail price} = \text{Original cost} + \text{Markup}$$

A store buys soccer balls for \$5.
The store's markup is 45%.

Step 1

Find the markup.

$$\$5 \times 0.45 = \$2.25$$

Step 2

ADD the markup to the original cost.

$$\$5 + \$2.25 = \$7.25$$

The retail price of a soccer ball is \$7.25.

Problem 2

Find the sale price.

$$\text{Sale price} = \text{Original price} - \text{Markdown}$$

The original price of a sweater is \$60.
On sale, the markdown is 20%.
Find the sale price.

Step 1

Find the markdown.

$$\$60 \times 0.20 = \$12$$

Step 2

SUBTRACT the markdown from the original price.

$$\$60 - \$12 = \$48$$

The sale price of the sweater is \$48.

1. Explain the difference between a markup and a markdown.

2. Explain the difference between a retail price and a sale price.

3. Write your own word problem using a markup or a markdown.

LESSON
5-3

Applications of Percent

Practice and Problem Solving: A/B

1. Complete the table.

Sale Amount	5% Sales Tax	Total Amount Paid
\$67.50		
\$98.75		
\$399.79		
\$1250.00		
\$12,500.00		

2. Complete the table.

Principal	Rate	Time	Interest Earned	New Balance
\$300	3%	4 years		
\$450		3 years	\$67.50	
\$500	4.5%		\$112.50	
	8%	2 years	\$108.00	

Solve.

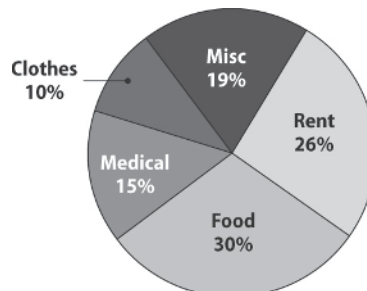
3. Joanna wants to buy a car. Her parents loan her \$5,000 for 5 years at 5% simple interest. How much will Joanna pay in interest?

4. This month Salesperson A made 11% of \$67,530. Salesperson B made 8% of \$85,740. Who made more commission this month? How much did that salesperson make?

5. Jon earned \$38,000 last year. He paid \$6,840 for entertainment. What percent of his earnings did Jon pay in entertainment expenses?

6. Nora makes \$3,000 a month. The circle graph shows how she spends her money. How much money does Nora spend on each category?

- a. rent _____
- b. food _____
- c. medical _____
- d. clothes _____
- e. miscellaneous _____



LESSON
5-3**Applications of Percent****Practice and Problem Solving: C**

1. Complete the table.

Sale Amount	Tax	Amount of Tax	Total Cost
\$49.95		\$4.00	\$53.95
	5%	\$6.43	
\$499.99	7.5%		
		\$103.96	\$2,702.96
\$12,499.00	7%		

2. Complete the table.

Principal	Annual Rate	Time	Interest Earned	New Balance
\$2,400		6 months		\$2,442.00
	4.9%	2 years	\$4.41	
\$9,460.12		5 years		\$12,061.65
\$3,923.87	2.2%		\$64.74	

Solve.

3. Jorge earns a 9% commission on all of his sales. He had sales of \$89,400 for the month. Harris works for a different company, and also sold \$89,400 for the month but made \$447 more than Jorge. What is Harris' commission rate?

4. Danielle wants to buy a video game. At the local Big Box store, it costs \$49.95. Danielle has a coupon for 10% off at the store, and she will pay a 6.5% sales tax. At an online store, the game is \$44.95, with \$4.00 shipping charge and no sales tax. Which purchase would be cheaper?

5. A clothing store ran advertisements for a special sale. The store's ads read "Buy one at regular price, get a second one for half price." Explain how the terms of the clothing store's sale are different from offering a 50% discount. Use \$100 as the regular price for the item to write your explanation.

LESSON
5-3**Applications of Percent****Practice and Problem Solving: D**

Complete the table to find the amount of sales tax to the nearest whole cent. The first one has been done for you.

1.

Sale Amount	5% Sales Tax
\$50	$0.05 \times \$50 = 2.5 = \2.50
\$120	
\$480	
\$2,240	
\$12,500	

Complete the table. The first one has been done for you.

2.

Principal	Rate	Time	Interest Earned
\$400	5%	2 years	\$40
\$950	10%	5 years	
\$50	4%	1 year	
\$1,000	8%	2 years	

Write the correct answer. The first one has been done for you.

3. Karl just had a birthday. Karl's age is now 50% of his uncle's age. Karl's uncle is 32 years old. How old is Karl?

$0.5 \times 32 = 16$, Karl is 16 years old.

4. The cost of Jacquie's gym membership is \$20 per month. Jacquie gets a 10% discount of the monthly cost in any month in which she refers a friend who also becomes a member. How much does Jacquie save in a month when one of her friends joins the gym?
-

5. Yesterday Tyler ate lunch at his local diner. The bill for his meal came to \$8.40, not including sales tax. Tyler wants to leave a 15% tip for his waiter. How much should his tip be?
-

LESSON

5-3

Applications of Percent**Reteach**

For any problem involving percent, you can use a simple formula to calculate the percent.

$$\text{amount} = \text{percent} \times \text{total}$$

The amount will be the amount of tax, tip, discount, or whatever you are calculating. Use the formula that has your unknown information before the equal sign.

For simple-interest problems, time is one factor. So, you must also include time in your formula.

$$\text{amount (interest)} = \text{total (principal)} \times \text{percent (rate)} \times \text{time}$$

A. Find the sale price after the discount.

Regular price = \$899

Discount rate = 20%

You know the total and the percentage. You don't know the discount amount. Your formula is:

$$\begin{aligned} \text{amount} &= \% \times \text{total} \\ &= 0.20 \times \$899 \\ &= \$179.80 \end{aligned}$$

The amount of discount is \$179.80.

The sale price is the original price minus the discount.

$$\$899 - \$179.80 = \$719.20$$

The sale price is \$719.20

B. A bank offers simple interest on a certificate of deposit. Jamie invests \$500 and after one year earns \$40 in interest. What was the interest rate on his deposit?

You know the total deposited—the principal. You know the amount earned in interest. You don't know the percentage rate of interest. Since the time is 1 year, your formula is:

$$\begin{aligned} \% &= \text{amount} \div \text{total} \\ &= \$40 \div \$500 \\ &= 0.08 \\ &= 8\% \end{aligned}$$

The interest rate is 8%.

Johanna purchases a book for \$14.95. There is a sales tax of 6.5%. How much is the final price with tax?

1. What is the total in this problem? _____
2. What is the percent? _____
3. Use the formula $\text{amount} = \text{total} \times \text{percent}$ to find the amount of the sales tax.

4. To find the final price, add the cost of the book to the amount of tax.

LESSON
5-3

Applications of Percent

Reading Strategies: Build Vocabulary

Sales tax is added to the price of an item or service. Sales tax is a percent of the purchase price. A sales tax of 6.5% means that all taxable items will have an additional 6.5% added to the total cost.

$$\text{sales tax rate} \times \text{sale price} = \text{sales tax}$$

The **total sale price** is computed by adding the sales tax to the cost of all the items purchased.

$$\text{sale price} + \text{sales tax} = \text{total sale price}$$

Find the amount of sales tax for each purchase to the nearest whole cent.

1. sale price: \$9,450

2. sale price: \$1,089

3. sale price: \$21,097

sales tax rate: 8%

sales tax rate: 6.25%

sales tax rate: 5.5%

Interest is the amount of money the bank pays to use your money, or the amount of money you pay the bank to borrow its money.

Principal is the amount of money you save or borrow from the bank.

Rate of interest is the percent rate on money you save or borrow.

Time is the number of years the money is saved or borrowed.

Answer each question.

4. You put \$800 in a savings account at 4% annual interest and leave it there for five years.

a. What is the principal? _____ b. What is the interest rate? _____

c. What is the amount of time the money will stay in the account?

Find out how much interest you would earn by using this formula:

Interest	=	Principal	×	Rate	×	Time	←	words
<i>i</i>	=	<i>p</i>	×	<i>r</i>	×	<i>t</i>	←	symbols
		\$800	×	4%	×	5		
		\$800	×	0.04	×	5	←	Change % to decimal.
		\$160					←	Multiply to solve.

5. To find out how much interest you will earn by keeping your money in a bank, what three things do you need to know?

LESSON
5-3

Applications of Percent

Success for English Learners

Problem 1

Find the tax on the sale.

HH Mart Welcome!	
DVD Player	\$148.50
DVD	\$15.99
CD	\$15.99
<u>Subtotal</u>	<u>\$180.48</u>
Tax (7.75%)	\$13.99

Add the cost of total purchases to get a subtotal.

Multiply the subtotal by the tax rate.

$$t = 7.75\% \times \$180.48$$

$$t = 0.0775 \times 180.78$$

$$t = \$13.99$$

So, the sales tax is \$13.99 on purchases.

Problem 2

Use this diagram to help you set up an equation for simple interest.

$$\text{Simple interest} = P \times r \times t$$

Principal

Rate

Time

Amount of money borrowed or invested

Interest rate written as a percent

Number of years money is borrowed or invested

Interest	=	\$14,500	×	7%	×	5 years
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$$I = 14,500 \times 0.07 \times 5$$

$$I = 5,075$$

Write 7% as $\frac{7}{100}$ and change to a decimal, 0.07.

The simple interest on \$14,500 invested or loaned for 5 years at 7% is \$5,075.

1. Jon invested \$6,200 for 6 years at 3%. Calculate the simple interest.

MODULE
5
Proportions and Percent
Challenge
Multi-Step Unit Conversion

Length (customary)	Length (metric)	Time	Metric-Customary
1 ft = 12 in.	1 cm = 10 mm	1 h = 60 m	1 in. = 2.54 cm
1 yd = 3 ft	1 m = 100 cm	1 m = 60 s	1 m = 39.37 in.
1 mi = 5,280 ft	1 km = 1,000 m	1 y = 365 d	

Using a method called “dimensional analysis,” you can convert any units for length, time, or speed using just the information in the chart.

This example shows converting 30 miles per hour (mi/h) to meters per second (m/s). Notice how the units cancel out.

$$\frac{30 \cancel{\text{mi}}}{1 \cancel{\text{h}}} \times \frac{5,280 \cancel{\text{ft}}}{1 \cancel{\text{mi}}} \times \frac{12 \cancel{\text{in.}}}{1 \cancel{\text{ft}}} \times \frac{1 \text{ m}}{39.37 \cancel{\text{in.}}} \times \frac{1 \cancel{\text{h}}}{60 \cancel{\text{min}}} \times \frac{1 \cancel{\text{min}}}{60 \text{ sec}}$$

Convert these units. Show your steps.

- Complete the calculation above to change 30 mi/h to meters per second.

- Change 2.3 km to inches.

- Change 67.3 ft/s to kilometers per hour.

Try these area and volume conversions. (Hint: Square factors for area; cube factors for volume.)

- A water tower is filled at 750 cubic feet per minute. Find the rate in gallons per hour. (1 gallon is about 0.134 cubic feet)

- A field is 130 ft by 274 ft. Find the area in square meters.

- A basement measures 9.6 m wide, 4.2 m tall, and 15.6 m long. Find the volume of space in the basement in cubic yards.

UNIT 2: Rates and Proportional Relationships

MODULE 4 Rates and Proportionality

LESSON 4-1

Practice and Problem Solving: A/B

- 2 eggs per batch
- 53 mph
- \$8/h
- 14 points per game
- \$0.20/oz
- $1\frac{3}{4}$ gal/h
- $\frac{1}{2}$ ft/min
- Food A: 200 cal/serving; Food B: 375 cal/serving; Food A has fewer calories per serving.

Practice and Problem Solving: C

- $\frac{1}{2}$ ac/h
- $2\frac{1}{5}$ mph
- $\frac{1}{80}$ of a wall
- $\frac{2}{9}$ oz
- $\frac{5\frac{1}{2} \text{ c}}{1\frac{9}{16} \text{ lb}} = \frac{88}{25} = \frac{3.52 \text{ c}}{1 \text{ lb}} = \frac{35.2 \text{ c}}{10 \text{ lb}}$; $35.2 > 35$, so there are more than 35 cups of flour in 10 lb of flour.
- Tank #1 is filling at a rate of 0.892857... gallons per hour while tank #2 is filling at a rate of $0.8\bar{3}$ gallons per hour. Since $0.892857... > 0.8\bar{3}$, tank #1 is filling faster.

Practice and Problem Solving: D

- 3; 3
- 45; 45
- \$9/h
- \$0.09/oz
- $\frac{\frac{3}{4} \text{ oz}}{3 \text{ h}} = \frac{3}{4} \div \frac{3}{1} = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4} \frac{\text{oz}}{1 \text{ h}}; \frac{1}{4} \text{ oz/h}$
- $\frac{3}{10}$ mi/min
- $\frac{150 \text{ cal}}{\frac{3}{4} \text{ serving}} = \frac{150}{1} \div \frac{3}{4} = \frac{150}{1} \times \frac{4}{3} = \frac{200 \text{ cal}}{1 \text{ serving}};$
200 cal/serving

Reteach

- $\frac{70 \text{ students}}{2 \text{ teachers}}$
- $\frac{3 \text{ books}}{2 \text{ mo}}$
- $\frac{\$52}{4 \text{ h}}$
- $\frac{28 \text{ patients}}{2 \text{ nurses}} = \frac{28 \div 2}{2 \div 2} = \frac{14 \text{ patients}}{1 \text{ nurse}}$
- $\frac{5 \text{ qt}}{2 \text{ lb}} = \frac{5 \div 2}{2 \div 2} = \frac{2.5 \text{ qt}}{1 \text{ lb}}$
- $\frac{3 \text{ oz}}{\frac{3}{4} \text{ c}} = 3 \div \frac{3}{4} = \frac{3}{1} \times \frac{4}{3} = \frac{4 \text{ oz}}{1 \text{ c}}$
- $\frac{3\frac{2}{3} \text{ ft}}{\frac{11}{60} \text{ h}} = 3\frac{2}{3} \div \frac{11}{60} = \frac{11}{3} \times \frac{60}{11} = \frac{20 \text{ ft}}{1 \text{ h}}$

Reading Strategies

- No; It does not compare values that have different units.
- Yes; It compares a number of yards to a number of seconds.

3. It compares miles to gallons.

4. Yes

5. No; $\frac{25 \text{ mi}}{1 \text{ gal}}$

6. No; $\frac{800 \text{ ft}^2}{1 \text{ h}}$

7. No; $\frac{2}{45} \text{ lb}$ or $\frac{8}{3} \text{ lb}$
 $\frac{1 \text{ min}}{1 \text{ min}}$ or $\frac{1 \text{ h}}{1 \text{ h}}$

Success for English Learners

1. 3 miles per hour or $\frac{3 \text{ mi}}{1 \text{ h}}$

2. $3\frac{3}{4}$ miles per hour or $\frac{3\frac{3}{4} \text{ mi}}{1 \text{ h}}$

3. Briana has the faster speed per hour.

LESSON 4-2

Practice and Problem Solving: A/B

1. a. yes

b. Sample answer: $c = 27t$

c. t

d. c

2. a. yes

b. Sample answer: $c = 4.35w$

c. w

d. c

3. not proportional

4. yes; Sample answers: $d = 40t$;
 $d = \text{distance}$; $t = \text{time}$

5. $k = \frac{1}{3}$; Sample answers: $b = \frac{1}{3}p$;

$b = \text{boxes}$; $p = \text{pens}$

6. $k = 6$; Sample answers: $m = 6p$;
 $m = \text{muffins}$; $p = \text{packs}$

7. a.

Days	1	2	3	4	5
Hours	24	48	72	96	120

b. yes

c. Sample answer: $h = 24d$ where d is the number of days and h is the number of hours

Practice and Problem Solving: C

1. a.

Number of Tickets	1	2	3	4	5
Total Cost (\$)	27	54	81	108	135

b. 27

c. Sample answer: $c = 27t$

2. 32

3. yes; Sample answers: $p = 35h$; h is number of hours; p is pages read

4. yes; Sample answers: $y = 6x$; x is number of ounces; y is grams of protein

5. yes; Sample answers: $c = 4.5w$; w is weight; c is total cost

6. no; You cannot write an equation for the pairs in the table as they are not proportional.

Practice and Problem Solving: D

1. a. yes

b. $y = 6x$

c. x

d. y

2. a. yes

b. $c = 3h$

c. h

d. c

3. yes; Sample answer: $c = 0.75w$;
 $w = \text{weight (oz)}$; $c = \text{total cost}$

4. not proportional

5. $k = \frac{1}{5}$; Sample answer: $b = \frac{1}{5}a$;

$a = \text{apples}$; $b = \text{bags}$

6. $k = 12$; Sample answer: $e = 12c$;
 $c = \text{cartons}$; $e = \text{eggs}$

Reteach

1. yes

2. $\frac{3}{1} = 3$; $\frac{6}{2} = 3$; $\frac{9}{3} = 3$; $\frac{12}{4} = 3$

3. Sample answer: $y = 3x$

4. 3

5. $y = 35x$

6. $y = 7x$

Reading Strategies

1. $\frac{3}{1} = 3$; $\frac{6}{2} = 3$; $\frac{9}{3} = 3$; $\frac{12}{4} = 3$

2. 3


3. yes

4. $\frac{35}{1}$

5. $\frac{4.35}{1}$

Success for English Learners

1.

	6	3	9	12	15
	2	1	3	4	5

2. 3

LESSON 4-3

Practice and Problem Solving: A/B

1.

Time (h)	2	4	5	9
Pay (\$)	16	32	40	72

Earnings are always 8 times the number of hours.

2.

Weight (lb)	2	3	6	8
Price (\$)	1.40	2.10	4.20	5.60

Cost is always 0.7 times the number of pounds.

- Not proportional; The line will not pass through the origin.
- Proportional; The line will pass through the origin.
- The car uses 2 gal of fuel to travel 40 mi.
- $y = 20x$, where x is the gallons of fuel used, y is the distance traveled (in miles), and k is the constant of proportionality
- The graph for the compact car would be steeper.

Practice and Problem Solving: C

- Employee B; Answers may vary. Sample answer: Employee A earns \$7.50 per hour, and employee B earns \$10 per hour, so employee B earns more money.
- Employee A: $15 \times \$7.50 = \112.50 ; employee B: $15 \times \$10.00 = \150.00
- Sample answer: $y = 8x$
- Company A: proportional because a graph comparing months of service and total cost will form a line passing through the origin; Company B: not proportional because the line formed will not pass through the origin
- Yes; $y = 2x$
- Sample answer: Graph the points and analyze the graph. The graph of a proportional relationship is a line that passes through the origin.

Practice and Problem Solving: D

- proportional; The cost is always 10 times the number of shirts.
- proportional; The number of crayons is always 50 times the number of boxes.
- proportional; The line will pass through the origin.
- not proportional; The line will not pass through the origin.
- $y = 6x$
- $y = 4x$
- $y = \frac{1}{3}x$
- Use the point (1, 8) to find the constant of proportionality, 8 or $\frac{8}{1}$

Reteach

- hours worked; pay (in dollars); Sample answer: (2, 14), $\frac{14}{2} = 7$; $y = 7x$
- number of students; cost of admission (in dollars); Sample answer: (12, 24), $\frac{24}{12} = 2$; $y = 2x$

Reading Strategies

1. number of glasses filled and ounces of juice needed
2. The ounces of juice needed is 8 times the number of glasses filled.
3. the number of glasses filled
4. the ounces of juice needed
5. 16 ounces of juice are needed to fill 2 glasses.
6. (3, 24)

Success for English Learners

1. Constant of proportionality = $\frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}}$; The constant of proportionality is the ratio of the change in y -values to the change in x -values.
2. No; only lines that pass through (0, 0) represent proportional relationships.

MODULE 4 Challenge

Sample answers are given. You may also wish to have students make sketches to show their graphing explorations.

1. direct: straight line through (0, 0), makes 45° angle
quadratic: curved line, starts at (0, 0), increases quickly as x increases
indirect: y starts really large, decreases to (1, 1), continues getting smaller and smaller
2. direct: quadrants I and III
quadratic: quadrants I and II
indirect: quadrants I and III; $y = \frac{k}{x}$ does not include the value $x = 0$, since this is undefined.
3. direct: slope of the line increases, it gets steeper
quadratic: the parabola gets skinnier
indirect: the curves pull away from the origin
4. direct: quadrants II and IV
quadratic: quadrants III and IV
indirect: quadrants II and IV; $y = \frac{k}{x}$ does not include the value $x = 0$, since this is undefined.

MODULE 5 Proportions and Percent

LESSON 5-1

Practice and Problem Solving: A/B

1. 25%
2. 150%
3. 200%
4. 122%
5. 71%
6. 53%
7. 45%
8. 75%
9. 62%
10. 90%
11. 17%
12. 19%
13. \$100
14. 128 bananas
15. 14 books
16. 65 companies
17. 12,600 miles
18. 639 points
19. 399 students
20. \$12.87; \$26.13
21. \$40.80
22. \$12,750

Practice and Problem Solving: C

1. a. 289
b. 332
2. a. 47% increase, 28% decrease, 50% increase
b. 20% increase, 17% increase, 10% increase
c. Rodrigo received a bonus in weeks 2 and 4. Samantha received a bonus in week 3.

d. Neither Rodrigo nor Samantha met the goal of increasing sales by 15% each week.

3. 2.7%

Practice and Problem Solving: D

- 40%
- 300%
- 90%
- 75%
- 81%
- 75%
- 33%
- 67%
- \$27.50
- 128 bananas
- 50 books
- 65 companies
- 420 students
- \$27.30

Reteach

- 14; 8; $\frac{14}{8}$; 175%
- 9; 90; $\frac{9}{90}$; 10%
- 75; 125; $\frac{75}{125}$; 60%
- 340; 400; $\frac{340}{400}$; 85%
- 25%
- 95%
- 80%
- 40%
- 200%
- 5%

Reading Strategies

- \$50
- decrease
- in the denominator (or bottom part) of the fraction

4. 25

5. 20

6. $\frac{20}{25} = 0.8 \times 100 = 80\%$; percent increase

Success for English Learners

- A percent increase is when the amount increases or goes up. A percent decrease is when the amount decreases or goes down.
- Sample answer: The height of a child from one year to the next.
- Retail is the price for the customer. Wholesale is the amount that the store bought the item for.
- wholesale price
- Answers will vary. Sample answer: Mr. Jiro buys a pack of T-shirts for \$4.95. He plans to sell them at an 80 percent increase. What is the selling price of each pack of T-shirts? ($\$4.95 \cdot 80 = \3.96 ; selling price: $\$4.95 + \$3.96 = \$8.91$.)

LESSON 5-2

Practice and Problem Solving: A/B

- \$0.30; \$1.80
- \$1.30; \$4.55
- \$2.40; \$12.00
- \$9.75; \$22.25
- \$42.90; \$120.90
- \$4.49; \$7.48
- \$57.20
- \$19.99
- \$35.70
- \$276.68
- 0.57c or 0.57
- $1 + 0.57c$ or $1.57c$
- \$70.65
- \$25.65

Practice and Problem Solving: C

- \$89.99
- \$30

3. 50%
4. \$90.75
5. \$113.44
6. \$76.00
7. $1.07c$
8. $1.02c$
9. Store B

Practice and Problem Solving: D

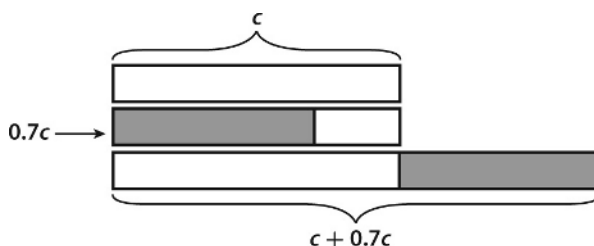
1. a. $0.40p$
b. $p + 0.4p$
c. \$78.40
d. \$22.40
2. \$6; \$36
3. \$3.50; \$13.50
4. \$10; \$50
5. \$58.50
6. \$21.35
7. \$26.25
8. \$276.25
9. $c + 0.4c$

Reteach

1. $\$45.00 + \$9.00 = \$54.00$
2. $\$7.50 + \$3.75 = \$11.25$
3. $\$1.25 + \$1.00 = \$2.25$
4. $\$21.70 + \$62.00 = \$83.70$
5. $\$150.00 - \$60.00 = \$90.00$
6. $\$18.99 - \$4.75 = \$14.24$
7. $\$95.00 - \$9.50 = \$85.50$
8. $\$75.00 - \$11.25 = \$63.75$
9. a. \$3.15
b. \$2.52

Reading Strategies

1–4.



$$\begin{aligned} \text{Retail price} &= \text{Original cost} + \text{markup} \\ &= c + 0.7c \\ &= 1.7c = 1.7(\$80) = \$136 \end{aligned}$$

1. the bar for the cost of a camera, c
2. the bar that shows the markup, 70% of c , or $0.7c$
3. the original cost plus the markup, $c + 0.7c$.
4. \$136

Success for English Learners

1. A markup is when the price increases or goes up. A markdown is when the price decreases or goes down.
2. The retail price is the original cost of an item plus a markup. The sales price is the original price of an item minus a markdown.
3. Answers will vary. Sample answer: A store buys shirts for \$15. The store's markup is 50%. What is the retail price? (\$22.50)

LESSON 5-3

Practice and Problem Solving: A/B

1.

Sale Amount	5% Sales Tax	Total Amount Paid
\$67.50	\$3.38	\$70.88
\$98.75	\$4.94	\$103.69
\$399.79	\$19.99	\$419.78
\$1250.00	\$62.50	\$1,312.50
\$12,500.00	\$625.00	\$13,125.00

2.

Principal	Rate	Time	Interest Earned	New Balance
\$300	3%	4 years	\$36.00	\$336.00
\$450	5%	3 years	\$67.50	\$517.50
\$500	4.5%	5 years	\$112.50	\$612.50
\$675	8%	2 years	\$108.00	\$783.00

3. \$1,250
4. salesperson A; \$7,428.30
5. 18%

6. a. \$780
- b. \$900
- c. \$450
- d. \$300
- e. \$570

Practice and Problem Solving: C

1.

Sale Amount	Tax	Amount of Tax	Total Cost
\$49.95	8%	\$4.00	\$53.95
\$128.60	5%	\$6.43	\$135.03
\$499.99	7.5%	\$37.50	\$537.49
\$2,599	4%	\$103.96	\$2,702.96
\$12,499	7%	\$874.93	\$13,373.93

2.

Principal	Rate	Time	Interest Earned	New Balance
\$2,400	3.5%	6 months	\$42.00	\$2,442.00
\$45.00	4.9%	2 years	\$4.41	\$49.41
\$9,460.12	5.5%	5 years	\$2,601.51	\$12,061.65
\$3,923.87	2.2%	9 months	\$64.74	\$3,988.61

3. Jorge earned \$8,046. Harris earned \$8,493. Harris' commission rate is 9.5%.
4. The total at Big Box store comes to \$47.88. The total online comes to \$48.95. It is cheaper at the Big Box store.
5. The first item is full price: \$100. The second item is half off: \$50. The total comes to \$150. A 50% discount on \$200 would be \$100.

Practice and Problem Solving: D

1.

Sale Amount	5% Sales Tax
\$50	$0.05 \times \$50 = 2.5 = \2.50
\$120	$0.05 \times \\$120 = \\6
\$480	$0.05 \times \\$480 = \\24
\$2,240	$0.05 \times \\$2,240 = \\112
\$12,500	$0.05 \times \\$12,500 = \\625

2.

Principal	Rate	Time	Interest Earned
\$400	5%	2 years	\$40
\$950	10%	5 years	\$475
\$50	4%	1 year	\$2
\$1,000	8%	2 years	\$160

3. $0.5 \times 32 = 16$; Karl is 16 years old.
4. $0.10 \times 20 = 2.0$; Jacque saves \$2 for referring a friend.
5. $0.15 \times 8.40 = 1.26$; Tyler's tip should be \$1.26.

Reteach

1. \$14.95
2. 6.5%
3. amount = $\$14.95 \times 6.5\% = \0.97
4. $\$14.95 + \$0.97 = \$15.92$

Reading Strategies

1. \$756
2. \$68.06
3. \$1,160.34
4. a. \$800
b. 4%
c. 5 years
5. principal, rate, and time

Success for English Learners

1. \$1,116

MODULE 5 Challenge

Possible solution steps are shown.

$$1. \frac{30 \times 5,280 \times 12 \text{ m}}{39.37 \times 60 \times 60 \text{ s}} = 13.41 \text{ m/s}$$

$$2. 2.3 \text{ km} \times \frac{1,000 \text{ m}}{1 \text{ km}} \times \frac{100 \text{ cm}}{1 \text{ m}} \times \frac{1 \text{ in.}}{2.54 \text{ cm}} = 90,551.2 \text{ in.}$$

$$3. \frac{67.3 \text{ ft}}{1 \text{ s}} \times \frac{12 \text{ in.}}{1 \text{ ft}} \times \frac{2.54 \text{ cm}}{1 \text{ in.}} \times \frac{1 \text{ m}}{100 \text{ cm}} \times \frac{1 \text{ km}}{1,000 \text{ m}} \times \frac{60 \text{ s}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ h}} = 73.85 \text{ km/h}$$

$$4. \frac{750 \text{ ft}^3}{1 \text{ min}} \times \frac{1 \text{ gal}}{0.134 \text{ ft}^3} \times \frac{60 \text{ min}}{1 \text{ h}} = 335,820.9 \text{ gal/h}$$

$$5. 130 \text{ ft} \times 274 \text{ ft} = 35,620 \text{ ft}^2; 35,620 \text{ ft}^2 \times \frac{(12)^2 \text{ in.}^2}{1 \text{ ft}^2} \times \frac{(2.54)^2 \text{ cm}^2}{1 \text{ in.}^2} \times \frac{1 \text{ m}^2}{(100)^2 \text{ cm}^2} = 3,309.2 \text{ m}^2$$

$$6. 9.6 \times 4.2 \times 15.6 = 628.992 \text{ m}^3; 628.992 \text{ m}^3 \times \frac{(39.37)^3 \text{ in.}^3}{1 \text{ m}^3} \times \frac{1 \text{ ft}^3}{(12)^3 \text{ in.}^3} \times \frac{1 \text{ yd}^3}{(3)^3 \text{ ft}^3} = 822.69 \text{ yd}^3$$